Imagining the interior

critical thinking on the pairing
of a cube and dodecahedron

Based on student exercises by Hans Schekper (glassgeometry.com)
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Objective: To teach students how to **see**,
how to **imagine**,
and how to **think**.

Feel free to pair whatever standards, competencies, and objectives fit your specific context. We have used variations on this project for Technology, Mathematics, and Arts competencies.

Technology *International Technology Education Association*	**Mathematics** *Common Core Mathematics*	**Arts** *National Core Arts Standards*
Nature of Technology 3: Students will develop an understanding of the relationships among technologies and the connections between technology and other fields of study **Technological World 11:** Student will develop the abilities to apply the design process	**Modeling** **Geometry: Modeling** ~ Apply geometric concepts in modeling situations **Geometry: Measurement and Dimensions** ~ Explain volume formulas and use them to solve problems ~ Visualize relationships between two-dimensional and three-dimensional objects	**Anchor Standard**: Generate and conceptualize artistic ideas and work. **VA:Cr1.1.HSI** Use multiple approaches to begin creative endeavors. **Anchor Standard**: Organize and develop artistic ideas and work. **VA:Cr2.1.HSI** Engage in making a work of art or design without having a preconceived plan.

Materials needed:
Prints of the two .STL files (4a, 5c, 6a and 7), paper, drawing materials. Note there are more pairings and files to allow for extension inquiries and observations beyond this one activity.

1. **Preprint the dodecahedron/cube pair** (4a). Do not tell the students anything about the shape.

2. **Have students write down what they see**- focusing on observation, try to come up with 10 points of observation.
   No conclusions- stick to observations. Some examples might be: "I see edges," "I see corners where 5 edges come together"

3. **Share the observations**- group shouts out their points with a class scribe
   Focus on the sequence of going from outside-in. Try to start with the surface before digging in to the interior.

4. **Allow some conjecture**- let the students define the dodecahedron, and possibly the cube- *Is it a cube? Is it just the outline of a cube? How are they sure of what they know?*
   Students will not know for sure whether the interior cube has solid faces or not. Allow them to challenge each other with their observations. How can they "prove" it is one or the other? *How many pentagons do you see? How many trapezoids (even though they don't exist)*

5. **Now imagine the cube has mirrored faces**. What would be going on with the mirrored **faces of the cube**? They look concave… how can that happen? Eventually the conclusion is that they're "reflection of the rooftops from the dodecahedron"

6. What could the **inside of the cube** look like based on the visual cues of the mirrored faces? Have students sketch and debate their ideas. Students should at this point be sharing and attempting to draw their ideas, using more precise geometric language.

7. **Share the print (5c)** of the cube with the inverted dodecahedral section removed. Imagine all of the cube faces were concave like this one. *Have students sketch what they believe this would look like.*

8. **Ask the students**, "Is there empty space inside the cube, or do the faces all touch flush? If there is space, what does it look like?" *Have the students sketch what they think it might look like.*

9. **Share the prints of spikey (7)***. How many students were close to this design? How did they go about **Spikey is our nickname for the cube with the inverted dodecahedral sections removed.*

10. **Show the inversion model again (6a)**- how the spikey print fits securely into the cube.

11. Ask to how they would **find the volume of spikey-** *not focusing on a formula, but an algorithm, a recipe.* Focus on the students' ability to reason the steps, not necessarily memorize the exact formulas. Have students share their various methods for communicating their recipes. If there's time, they can begin to test their communication with each other- seeing how clear their communication is.

12. Have students design their own polygon pair with an unknown interior. They can sketch or research, and figure out how to print it themselves.

Observations/Conclusions: *How do you know what you know?* In the spikey exercise think of an example of a statement you or someone else made, where you had to defend how you knew it (i.e. "it's a dodecahedron, they're mirrors") and detail how you had to back up your thinking. What kind of evidence was good enough to defend your thinking? When are conclusions ok to use, and when do you need more concrete observations?

You need to "back up" to move forward.

Breaking down Math Problems: How do you tackle a problem you can't easily see?

- How do you find the area of a pentagon? *Break it down into triangles*
- What does spikey look like? *Try feeling it out, literally*
- What's the volume of spikey? *Cubes and dodecahedrons?*
- How do you find the edges of the cubes? *Use the dodecahedron's diagonals*…

Writing: What did you learn in doing this exercise?
Writing or Sketch: Can you find any more regular shapes in cross sections?